On the Dynamics of a "Curved Ball."

By Ormond Stone, *Cincinnati, O.*

In his paper "On the Lateral Deviation of Spherical Projectiles,"* Professor Eddy has made a mistake which destroys the force of his argument. In the equation (7)

$$\cos\psi = \sin\delta \sin\theta \cos\phi - \cos\delta \cos\theta\,,$$

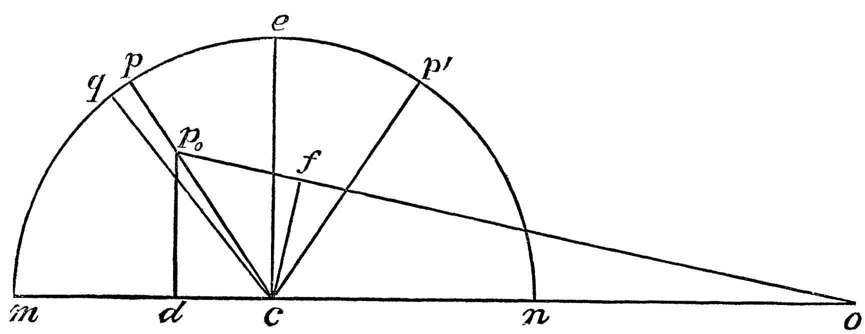
ψ is the arc of a great circle joining dS to m and not that joining it to n'. The cosine of the arc joining dS to n' is

$$\sin\delta \sin\theta \cos\phi + \cos\delta \cos\theta\,.$$

If a plane be passed through dS, m and o, it is manifest that the pressure dP will lie in this plane, and that the component acting from c toward o will be proportional to the cosine of the angle at c subtended by the arc joining dS to m.

"Again," says Professor Eddy, "it appears from the interpretation given to ψ, . . . that, so long as $\theta < 90°$, more than half the elements dS along this ring between pp' and qq' are within 90° of n', and hence the largest positive value of $\cos\psi$ numerically exceeds its largest negative value." Now, if p be between e and g, all the elements dS along the ring are within 90° of n', although the pressure upon each of these elements in the direction co is negative.

Equation (8) is somewhat complicated, but can be readily integrated for special cases. The following solution of the problem is suggested as preferable. Professor Eddy's nomenclature, etc., are retained as far as practicable.



* This Journal, Vol. II, pp. 85–88.

"In the figure let c be the center of a spherical projectile whose radius is a, and let *men* be" one-half of "the great circle of the sphere which lies in a horizontal plane. Let us disregard the vertical component of the motion of the projectile; and let c have a horizontal motion of translation, at the instant under consideration, towards e. Also, let the projectile have a motion of rotation about a vertical axis through c in a right-handed direction, *i. e.* from m to e. The motions of translation and rotation, whatever be their relative velocities, can be combined, as is well known, into a single motion of rotation about an instantaneous axis parallel to the vertical axis of rotation through c. This instantaneous axis must intersect the diameter mn, which is perpendicular to the direction of translation ce at some point, as o. Let the instantaneous axis through o be called the axis of z. Also, let the distance oc be designated by the letter b."

Let r be the distance of any element dS of the surface of the sphere from the axis of z. Pass a vertical plane through c, cutting the hemisphere *men* in a semicircle whose projection is pc, and similarly pass a second plane qc, making an infinitesimal angle $d\mu = pcq$ with pc, and let $\delta = fcp$, $fco = dp_0o = \theta$, $ecp = \mu$, $cp_0 = a \cos \nu$;

$$\therefore \quad cf = a \cos \nu \cos \delta = b \cos \theta ,$$

$$dp_0 = a \cos \nu \cos \mu = r \cos \theta ;$$

$$\therefore \qquad \cos \delta = \frac{b}{r} \cos \mu .$$

Since z is the instantaneous axis of rotation,

$$v = cr ,$$

where v is the velocity of any element of the surface of the sphere and c is a constant.

Let dS be the quadrilateral element of the spherical surface included between the semicircles pc and qc, making an angle $d\mu$ with one another, and two small circles parallel with the horizon having a difference in altitude of $d\nu$ measured in arc on the surface of the sphere. Let p_0 be the projection of dS on the horizontal plane; then dS is ultimately a rectangle the length of whose sides are $ad\nu$ and $a \cos \nu \; d\mu$;

$$\therefore \quad dS = a^2 \cos \nu \; d\mu \; d\nu .$$

If we assume that the pressure dP on dS is toward c and proportional to v^n, where n is a constant > 1, and to $\cos \delta \; dS$ the cross section of the stream of

air which dS meets in its motion, we have

$$dP = c'v^n \cos\delta \, dS = mr^{n-1} \cos\mu \cos\nu \, d\mu \, d\nu,$$

where c' and m are constants and $m = a^2bc^nc'$.

The component dX of dP, acting in the direction co, is $\sin\mu \cos\nu \, dP$;

$$\therefore \; dX = r^{n-1} dQ$$

where $dQ = \frac{1}{2} m \sin 2\mu \cos^2\nu \, d\mu \, d\nu$.

Also, if r' be the distance from the axis of z of an element of the surface dS' having an azimuth $-\mu = ecp'$ and altitude ν, the component of the pressure dP' in the direction co is

$$dX' = -r'^{n-1} dQ;$$

$$\therefore \; d(X + X') = (r^{n-1} - r'^{n-1}) \, dQ.$$

Hence, since r is greater than r', $d(X + X')$ is positive, *i. e.* the deviating pressure acting upon the two elements dS and dS' is from c towards o.

The normal pressures acting upon dS and dS' are evidently greater than the average pressure of the atmosphere. On the other hand, the motion of the ball causes a diminution of pressure upon the elements dS'' and dS''' whose altitude is the same as that of dS and dS', but whose azimuths are $180° - \mu$ and $180° + \mu$. Since the velocity of dS'' is greater than that of dS''', the diminution of the pressure upon dS'' is greater than that upon dS'''. Nevertheless, the increase of pressure in front, caused by an increase in velocity of a body moving through a homogeneous elastic fluid is always greater than the corresponding decrease behind;

$$\therefore \; dP - dP' > dP'' - dP''';$$

consequently, since $d(X + X')$ is positive, $d(X + X' + X'' + X''')$ is also positive; or, in other words, the total lateral pressure upon the four points in question, and hence by integration the total lateral pressure upon the projectile is from c toward o.